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On $osp(2|2)$ conformal field theories

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Abstract

We study the conformal field theories corresponding to current superalgebras $osp(2|2)_k^{(1)}$ and $osp(2|2)_k^{(2)}$. We construct the free field realizations, screen currents and primary fields of these current superalgebras at general level k . All the results for $osp(2|2)_k^{(2)}$ are new, and the results for the primary fields of $osp(2|2)_k^{(1)}$ also seem to be new. Our results are expected to be useful in the supersymmetric approach to Gaussian disordered systems such as the random bond Ising model and the Dirac model.

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1. Introduction

In recent years, disordered systems have attracted much attention in both theoretical and condensed matter physics communities [1–8]. In particular, the application of the supersymmetric method [9] to Gaussian disordered systems has revealed that the relevant algebras are current superalgebras with zero superdimension [10–14]. Such superalgebras have equal number of bosonic and fermionic generators. This ensures that the Virasoro algebra constructed from the super currents has vanishing central charge: a necessary condition for the description of disordered systems. The conformal field theory derived from such a current superalgebra potentially contains primary fields with negative conformal dimensions so that the theory is non-unitary. The non-unitarity makes the conformal field theory non-trivial even though it has a vanishing central charge.

Our aim in this paper is to provide some algebraic backgrounds which are expected to be useful in the study of the random bond Ising model and the two-species Dirac model with a random $sl(2)$ -gauge potential. Namely, we investigate the conformal field theories based on the current superalgebras $osp(2|2)^{(1)}$ and $osp(2|2)^{(2)}$ at general level k . We derive the free field representations and screen currents of these two algebras. Primary fields corresponding to both typical and atypical representations are constructed explicitly and their operator product

expansions (OPEs) with currents are presented. In the case of $osp(2|2)^{(1)}$, there exists an infinite family of negative dimensional primary operators, and for the case of $osp(2|2)^{(2)}$ all primary fields have zero conformal dimensions. All results for $osp(2|2)_k^{(2)}$ are new, and the explicit results for the primary fields of $osp(2|2)_k^{(1)}$ also seem to be new. As for the free field realizations and screen currents of $osp(2|2)_k^{(1)}$, similar results have also been obtained in [15–18], though based on different approaches and conventions. We used a slightly more straightforward approach by means of super coherent states. The free field realizations of the currents are needed in order to find all representations (i.e. primary fields) of the current superalgebras.

This paper is organized as follows. In section 2, we set our convention. As is well known, free field realization is a common approach used in both conformal field theories and representation theory of current algebras [19–23]. So in section 3, we describe our construction of the free field representations and screen currents. In section 4, we construct the primary fields corresponding to both typical and atypical representations of the current superalgebras. We conclude in section 5.

2. Notation

It is well known that unlike a purely bosonic algebra a superalgebra admits different Weyl inequivalent choices of simple root systems, which correspond to inequivalent Dynkin diagrams. In the case of $osp(2|2)$, one has two choices of simple roots which are unrelated by Weyl transformations: a system of fermionic and bosonic simple roots (i.e. the so-called standard basis) or a purely fermionic system of simple roots (that is the so-called non-standard basis). So it is useful to obtain results in the two different bases for different physical applications. Moreover, it seems that only in the non-standard basis could $osp(2|2)$ be twisted to give $osp(2|2)^{(2)}$.

2.1. $osp(2|2)^{(1)}$ in the standard basis

Let E (F) and e (f) be the generators corresponding to the even and odd simple roots of $osp(2|2)$ in the standard (distinguished) basis, respectively. Let \bar{e} , \bar{f} be the odd non-simple generators. They satisfy the following (anti-)commutation relations:

$$\begin{aligned}
 [E, F] &= H & [H, E] &= 2E & [H, F] &= -2F \\
 \{e, f\} &= -\frac{1}{2}(H - H') & [H, e] &= -e & [H, f] &= f \\
 [H', e] &= -e & [H', f] &= f \\
 [E, e] &= \bar{e} & [F, f] &= \bar{f} \\
 \{\bar{e}, \bar{f}\} &= -\frac{1}{2}(H + H') \\
 \{e, \bar{f}\} &= -F & \{\bar{e}, f\} &= E \\
 [E, \bar{f}] &= f & [F, \bar{e}] &= e \\
 [H, \bar{e}] &= \bar{e} & [H, \bar{f}] &= -\bar{f} \\
 [H', \bar{e}] &= -\bar{e} & [H', \bar{f}] &= \bar{f}.
 \end{aligned} \tag{2.1}$$

All other (anti-)commutators are zero. The quadratic Casimir is given by

$$C_2 = \frac{1}{2}(H(H+2) - H'(H'+2)) + 2fe - 2\bar{f}\bar{e} + 2FE. \tag{2.2}$$

This quadratic Casimir is useful in the following to construct the energy–momentum tensor.

The current superalgebra $osp(2|2)_k^{(1)}$ in the standard basis can be written as

$$J_A(z)J_B(w) = k \frac{\text{str}(AB)}{(z-w)^2} + f_{AB}^C \frac{J_C(w)}{(z-w)} \tag{2.3}$$

where f_{AB}^C are structure constants related to generators A, B and C , which can be read off from the above (anti-)commutation relations.

2.2. $osp(2|2)^{(1)}$ in the non-standard basis

In the non-standard basis, simple roots of $osp(2|2)$ are all fermionic. Let e, f, \bar{e}, \bar{f} be the generators corresponding such fermionic simple roots, and let E, F be the non-simple generators. They obey the (anti-)commutation relations:

$$\begin{aligned} \{e, f\} &= -\frac{1}{2}(H - H') & [H, e] &= e & [H, f] &= -f \\ [H', e] &= e & [H', f] &= -f \\ [H, \bar{e}] &= \bar{e} & [H, \bar{f}] &= -\bar{f} \\ [H', \bar{e}] &= -\bar{e} & [H', \bar{f}] &= \bar{f} \\ \{\bar{e}, \bar{f}\} &= -\frac{1}{2}(H + H') \\ \{e, \bar{e}\} &= E & \{\bar{f}, f\} &= -F \\ [E, F] &= H & [H, E] &= 2E & [H, F] &= -2F \\ [E, f] &= \bar{e} & [F, e] &= \bar{f} \\ [E, \bar{f}] &= e & [F, \bar{e}] &= f. \end{aligned} \tag{2.4}$$

All other (anti-)commutators are zero, and the quadratic Casimir is

$$C_2 = \frac{1}{2}(H^2 - H'^2) - 2fe - 2\bar{f}\bar{e} + 2FE. \tag{2.5}$$

The current superalgebra $osp(2|2)_k^{(1)}$ in the non-standard basis has the similar form as (2.3) except that f_{AB}^C are now derived from (2.4).

2.3. Twisted superalgebra $osp(2|2)^{(2)}$

Let us start with some basics of twisted affine algebras [30]. Let g be a simple finite-dimensional Lie algebra and σ be an automorphism of g satisfying $\sigma^r = 1$ for a positive integer r , then g can be decomposed into the form: $g = \bigoplus_{j=0}^{r-1} g_j$, where g_j is the eigenspace of σ with eigenvalue $e^{2j\pi i/r}$, and $[g_i, g_j] \subset g_{(i+j) \bmod r}$, then r is called the order of the automorphism.

Here we only consider the simplest twisted affine superalgebra $osp(2|2)^{(2)}$ so that $g = osp(2|2)$ and $r = 2$. We can write

$$osp(2|2) = g_0 \oplus g_1 \tag{2.6}$$

where $g_0 = osp(1|2)$ is a fixed point sub-superalgebra under the automorphism, while g_1 is a three-dimensional representation of g_0 , g_0 and g_1 satisfy $[g_i, g_j] \subset g_{(i+j) \bmod 2}$. We denote the

basis of g_0 by e, f, E, F, H and the basis for g_1 by e', f, H' . The commutation relations of $osp(2|2)$ in this basis are

$$\begin{aligned}
 [E, F] &= H & [H, E] &= 2E & [H, F] &= -2F \\
 \{e, e\} &= 2E & \{f, f\} &= -2F & \{e, f\} &= H \\
 [E, f] &= -e & [F, e] &= -f & & \\
 [H, e] &= e & [H, f] &= -f & & \\
 \{e', e'\} &= -2E & \{f', f'\} &= 2F & \{e', f'\} &= H \\
 [H', e'] &= -e & [H', f'] &= -f & & \\
 \{e', f\} &= H' & \{f', e\} &= H' & & \\
 [H, e'] &= e' & [H, f'] &= -f' & & \\
 [E, f'] &= e' & [F, e'] &= f' & & \\
 [H', e] &= -e' & [H', f] &= f'. & &
 \end{aligned} \tag{2.7}$$

All other (anti-)commutators are zero, and the quadratic Casimir is

$$C_2 = \frac{1}{2}(H^2 - H'^2) + 2fe + 2f'e' + 2FE. \tag{2.8}$$

The current superalgebra $osp(2|2)_k^{(2)}$ reads

$$J_A(z)J_B(w) = k \frac{\text{str}(AB)}{(z-w)^2} + f_{AB}^C \frac{J_C(w)}{z-w} \tag{2.9}$$

where f_{AB}^C are read off from (2.7).

3. Wakimoto realizations and screen currents

In this section, we examine the free field realizations of $osp(2|2)_k^{(1)}$ and $osp(2|2)_k^{(2)}$ and their screen currents. The results for $osp(2|2)_k^{(2)}$ are new. For $osp(2|2)_k^{(1)}$ similar results have also been obtained in [15–18] using different approaches. Let us remark that free field realizations of affine $osp(2|2)$ at $k = 1$ have also been constructed in [24, 25]. (Note: Ludwig used a different convention for $osp(2|2)_k^{(1)}$ in [25]; $k = -2$ in his convention is equivalent to $k = 1$ in our convention.)

To obtain free field realizations we first construct Fock space representations of $osp(2|2)$ corresponding to the bases given in section 2. Let E_α denote the raising generators of $osp(2|2)$. A highest weight state $|P, Q, P\rangle$ of $osp(2|2)$ is defined by

$$E_\alpha |P, Q, P\rangle = 0 \quad H |P, Q, P\rangle = P |P, Q, P\rangle \quad H' |P, Q, P\rangle = Q |P, Q, P\rangle. \tag{3.1}$$

3.1. $osp(2|2)_k^{(1)}$ in the standard basis

Let $A_{st} = xF + \theta f + \bar{\theta} \bar{f}$ be an operator in the standard basis of $osp(2|2)$, where x is bosonic coordinate, θ and $\bar{\theta}$ are fermionic coordinates. The action of $e^{A_{st}}$ on the highest weight state $|p, q, p\rangle$ generates a coherent state of $osp(2|2)$. Write

$$b_g e^{A_{st}} |p, q, p\rangle = D_g e^{A_{st}} |p, q, p\rangle \quad f_g e^{A_{st}} |p, q, p\rangle = d_g e^{A_{st}} |p, q, p\rangle \tag{3.2}$$

for bosonic generators b_g and fermionic generators f_g of $osp(2|2)$. Here D_g, d_g are the corresponding differential operators. Using the Baker–Campbell–Hausdorff (BCH) formula

and $osp(2|2)$ commutation relations, we obtain

$$\begin{aligned}
 d_{\bar{f}} &= \partial_{\bar{\theta}} \\
 d_f &= \partial_{\theta} + \frac{1}{2}x\partial_{\bar{\theta}} \\
 D_F &= \partial_x - \frac{1}{2}\theta\partial_{\bar{\theta}} \\
 D_H &= p + 2x\partial_x - \theta\partial_{\theta} + \bar{\theta}\partial_{\bar{\theta}} \\
 D_{H'} &= q - \theta\partial_{\theta} - \bar{\theta}\partial_{\bar{\theta}} \\
 d_e &= -\frac{1}{2}(p-q)\theta - \bar{\theta}\partial_x - \frac{1}{2}\theta x\partial_x - \frac{1}{2}\theta\bar{\theta}\partial_{\bar{\theta}} \\
 D_E &= -px - \bar{\theta}\partial_{\theta} - x^2\partial_x - \frac{1}{2}x\bar{\theta}\partial_{\bar{\theta}} + \frac{1}{2}x\theta\partial_{\theta} - \frac{1}{4}x^2\theta\partial_{\bar{\theta}} \\
 d_{\bar{e}} &= -\frac{1}{2}(p+q)\bar{\theta} - \frac{1}{4}(3p-q)x\theta - \bar{\theta}x\partial_x + \bar{\theta}\theta\partial_{\theta} - \frac{1}{2}\theta x^2\partial_x.
 \end{aligned}
 \tag{3.3}$$

It is straightforward to prove that the above differential operators satisfy the algebraic relations of $osp(2|2)$ in the standard basis.

We now use the differential operator representation (3.3) to find the Wakimoto realization of $osp(2|2)_k^{(1)}$ in terms of one bosonic β - γ pair, two fermionic b - c type systems and two free scalar fields. These free fields have the following OPEs:

$$\begin{aligned}
 \beta(z)\gamma(w) &= -\gamma(z)\beta(w) = -\frac{1}{z-w} & \psi(z)\psi^{\dagger}(w) &= \psi^{\dagger}(z)\psi(w) = -\frac{1}{z-w} \\
 \bar{\psi}(z)\bar{\psi}^{\dagger}(w) &= \bar{\psi}^{\dagger}(z)\bar{\psi}(w) = -\frac{1}{z-w} & \phi(z)\phi(w) &= -\ln(z-w) = \phi'(z)\phi'(w).
 \end{aligned}
 \tag{3.4}$$

The free field realization of $osp(2|2)^{(1)}$ in the standard basis is obtained by the following substitution:

$$\begin{aligned}
 d_{f(\bar{f})} &\rightarrow j_{e(\bar{e})}(z) & d_{e(\bar{e})} &\rightarrow j_{f(\bar{f})}(z) & D_F &\rightarrow J_E(z) \\
 D_E &\rightarrow J_F(z) & D_{H(H')} &\rightarrow J_{H(H')}(z) & \partial_x &\rightarrow \beta(z) \\
 x &\rightarrow \gamma(z) & \theta(\bar{\theta}) &\rightarrow \psi(z)(\bar{\psi}(z)) \\
 \partial_{\theta(\bar{\theta})} &\rightarrow \psi^{\dagger}(z)(\bar{\psi}^{\dagger}(z)) & p &\rightarrow i\alpha_+\partial\phi(z) & q &\rightarrow i\alpha_+\partial\phi'(z)
 \end{aligned}
 \tag{3.5}$$

in the differential operator realization (3.3) and a subsequent addition of anomalous terms linear in $\partial\psi(z)$, $\partial\gamma(z)$ or $\partial\bar{\psi}(z)$ in currents $j_{f(\bar{f})}(z)$ and $J_F(z)$. The result is

$$\begin{aligned}
 j_{\bar{e}}(z) &= \bar{\psi}^{\dagger}(z) \\
 j_e(z) &= -\psi^{\dagger}(z) - \frac{1}{2}\gamma(z)\bar{\psi}^{\dagger}(z) \\
 J_E(z) &= \beta(z) - \frac{1}{2}\psi(z)\bar{\psi}^{\dagger}(z) \\
 J_H(z) &= i\alpha_+\partial\phi(z) + 2\beta(z)\gamma(z) - \psi(z)\psi^{\dagger}(z) + \bar{\psi}(z)\bar{\psi}^{\dagger}(z) \\
 J_{H'}(z) &= \alpha_+\partial\phi'(z) - \psi(z)\psi^{\dagger}(z) - \bar{\psi}(z)\bar{\psi}^{\dagger}(z) \\
 j_f(z) &= -\frac{1}{2}\alpha_+(i\partial\phi(z) - \partial\phi'(z))\psi(z) - \beta(z)\bar{\psi}(z) - \frac{1}{2}\beta(z)\gamma(z)\psi(z) \\
 &\quad - \frac{1}{2}\bar{\psi}(z)\bar{\psi}^{\dagger}(z)\psi(z) + (k + \frac{1}{2})\partial\psi(z) \\
 J_F(z) &= -i\alpha_+\partial\phi(z)\gamma(z) - \beta(z)\gamma^2(z) - \bar{\psi}(z)\psi^{\dagger}(z) + \frac{1}{2}\gamma(z)(\psi(z)\psi^{\dagger}(z) + \bar{\psi}(z)\bar{\psi}^{\dagger}(z)) \\
 &\quad - \frac{1}{4}\gamma^2(z)\psi(z)\bar{\psi}^{\dagger}(z) - (k - \frac{1}{2})\partial\gamma(z) \\
 j_{\bar{f}}(z) &= \frac{1}{2}\alpha_+(i\partial\phi(z) + \partial\phi'(z))\bar{\psi}(z) + \frac{1}{4}\alpha_+(3i\partial\phi(z) - \partial\phi'(z))\gamma(z)\psi(z) \\
 &\quad + \beta(z)\gamma(z)\bar{\psi}(z) - \psi(z)\psi^{\dagger}(z)\bar{\psi}(z) + \frac{1}{2}\beta(z)\gamma^2(z)\psi(z) \\
 &\quad + k\partial\bar{\psi}(z) + \frac{1}{2}(k-1)\psi(z)\partial\gamma(z) - \frac{1}{2}(k+1)\gamma(z)\partial\psi(z)
 \end{aligned}
 \tag{3.6}$$

where $\alpha_+ = \sqrt{2k+2}$, and normal ordering is implied in the expressions.

The energy–momentum tensor is obtained by the Sugawara construction. Due to singularities which arise when multiplying currents at the same point we need to consider a regularization to remove such divergences. We use the usual point-splitting regularization where singular parts appearing in the OPEs of the currents are subtracted. This is equivalent to defining the normal ordered product of two fields $A(z)$ and $B(z)$ by

$$: AB : (z) \equiv \oint_w \frac{dz}{2\pi i} \frac{A(z)B(w)}{z-w}. \quad (3.7)$$

In the present case, the Sugawara energy–momentum tensor is given by

$$T_{st}(z) = \frac{1}{2(k+1)} : \left(\frac{1}{2} J_H(z) J_H(z) - \frac{1}{2} J_{H'}(z) J_{H'}(z) + J_E(z) J_F(z) + J_F(z) J_E(z) \right. \\ \left. - j_e(z) j_f(z) + j_f(z) j_e(z) + j_{\bar{e}}(z) j_{\bar{f}}(z) - j_{\bar{f}}(z) j_{\bar{e}}(z) \right) : . \quad (3.8)$$

By means of the free field representation of the currents, we get

$$T_{st}(z) = -\beta(z) \partial \gamma(z) + \psi^\dagger(z) \partial \psi(z) + \bar{\psi}^\dagger(z) \partial \bar{\psi}(z) \\ + \frac{1}{2} ((i\partial\phi(z))^2 + (\partial\phi'(z))^2) - \frac{1}{\alpha_+} (i\partial^2\phi(z) - \partial^2\phi'(z)). \quad (3.9)$$

The energy–momentum tensor satisfies the OPE,

$$T_{st}(z) T_{st}(w) = \frac{2T_{st}(w)}{(z-w)^2} + \frac{\partial T_{st}(w)}{z-w}. \quad (3.10)$$

So the Virasoro central charge of the theory is zero.

An important object in the free field approach is screening current. Screening currents are primary fields with conformal dimension 1, and their integrations give the screening charges. They commute with the affine currents up to a total derivative. These properties ensure that screening charges may be inserted into correlators while the conformal or affine ward identities remain intact. For the present case, the screening currents of first kind are found to be

$$s_{s,1}(z) = \left(\psi^\dagger(z) - \frac{1}{2} \gamma(z) \bar{\psi}^\dagger(z) \right) \exp \left\{ \frac{1}{\alpha_+} (i\phi(z) - \phi'(z)) \right\} \\ s_{s,2}(z) = \left(\beta(z) + \frac{1}{2} \psi(z) \bar{\psi}^\dagger(z) \right) \exp \left\{ -\frac{2}{\alpha_+} i\phi(z) \right\} \quad (3.11)$$

and the screening current of second kind is

$$s_{II}(z) = \left(\beta^{-k-1}(z) - \frac{k+1}{2} \beta^{-k-2}(z) \psi(z) \bar{\psi}^\dagger(z) \right) \exp\{\alpha_+ i\phi(z)\}. \quad (3.12)$$

Similar results to (3.6), (3.11), (3.12) have also been obtained in [17, 18] using different approaches.

3.2. $osp(2|2)_k^{(1)}$ in the non-standard basis

In the non-standard basis, the action of the operator $e^{A_{nst}}$ with $A_{nst} = xF + \theta f + \bar{\theta} \bar{f}$ on the highest weight state $|p, q, p\rangle$ generates a coherent state of $osp(2|2)$, where x is bosonic coordinate, θ and $\bar{\theta}$ are fermionic coordinates. Denote the action of $osp(2|2)$ generators on this coherent state by

$$b_g e^{A_{nst}} |p, q, p\rangle = D_g e^{A_{nst}} |p, q, p\rangle \quad f_g e^{A_{nst}} |p, q, p\rangle = d_g e^{A_{nst}} |p, q, p\rangle \quad (3.13)$$

where b_g, f_g are the bosonic and fermionic generators of $osp(2|2)$, respectively, and D_g, d_g are the corresponding differential operators. After some algebraic manipulations, we find

$$\begin{aligned}
 d_{\bar{f}} &= \partial_\theta - \frac{1}{2}\bar{\theta}\partial_x \\
 d_f &= \partial_{\bar{\theta}} - \frac{1}{2}\theta\partial_x \\
 D_F &= \partial_x \\
 D_H &= p + 2x\partial_x + \theta\partial_\theta + \bar{\theta}\partial_{\bar{\theta}} \\
 D_{H'} &= q + \theta\partial_\theta - \bar{\theta}\partial_{\bar{\theta}} \\
 d_e &= -\frac{1}{2}(p-q)\theta + x\partial_{\bar{\theta}} - \frac{1}{2}\theta x\partial_x - \frac{1}{2}\theta\bar{\theta}\partial_{\bar{\theta}} \\
 d_{\bar{e}} &= -\frac{1}{2}(p+q)\bar{\theta} + x\partial_\theta - \frac{1}{2}\bar{\theta}\theta\partial_\theta - \frac{1}{2}\bar{\theta}x\partial_x \\
 D_E &= -px + \frac{1}{2}q\theta\bar{\theta} - x^2\partial_x - x\bar{\theta}\partial_{\bar{\theta}} - x\theta\partial_\theta.
 \end{aligned} \tag{3.14}$$

It is easy to show that the above differential operators give a realization of $osp(2|2)$ in the non-standard basis.

With the help of the differential operator representation (3.14) and by a substitution similar to (3.5) and an addition of suitable anomalous terms, we find the free field realization of $osp(2|2)_k^{(1)}$ in the non-standard basis

$$\begin{aligned}
 J_E(z) &= \beta(z) \\
 j_e(z) &= \psi^\dagger(z) - \frac{1}{2}\beta(z)\bar{\psi}(z) \\
 j_{\bar{e}}(z) &= \bar{\psi}^\dagger(z) - \frac{1}{2}\beta(z)\psi(z) \\
 J_H(z) &= i\alpha_+\partial\phi(z) + 2\beta(z)\gamma(z) + \psi(z)\psi^\dagger(z) + \bar{\psi}(z)\bar{\psi}^\dagger(z) \\
 J_{H'}(z) &= \alpha_+\partial\phi'(z) + \psi(z)\psi^\dagger(z) - \bar{\psi}(z)\bar{\psi}^\dagger(z) \\
 j_f(z) &= \frac{1}{2}\alpha_+(i\partial\phi(z) - \partial\phi'(z))\psi(z) - \gamma(z)\bar{\psi}^\dagger(z) + \frac{1}{2}\beta(z)\gamma(z)\psi(z) \\
 &\quad + \frac{1}{2}\psi(z)\bar{\psi}(z)\bar{\psi}^\dagger(z) + (k + \frac{1}{2})\partial\psi(z) \\
 j_{\bar{f}}(z) &= \frac{1}{2}\alpha_+(i\partial\phi(z) + \partial\phi'(z))\bar{\psi}(z) - \gamma(z)\psi^\dagger(z) + \frac{1}{2}\beta(z)\gamma(z)\bar{\psi}(z) \\
 &\quad + \frac{1}{2}\bar{\psi}(z)\psi(z)\psi^\dagger(z) + (k + \frac{1}{2})\partial\bar{\psi}(z) \\
 j_F(z) &= -\alpha_+i\partial\phi(z)\gamma(z) + \frac{1}{2}\alpha_+\partial\phi'(z)\psi(z)\bar{\psi}(z) - \beta(z)\gamma^2(z) \\
 &\quad + \gamma(z)(\psi(z)\psi^\dagger(z) + \bar{\psi}(z)\bar{\psi}^\dagger(z)) - k\partial\gamma(z) \\
 &\quad + \frac{1}{2}(k+1)(\bar{\psi}(z)\partial\psi(z) + \psi(z)\partial\bar{\psi}(z)).
 \end{aligned} \tag{3.15}$$

The energy–momentum tensor in the non-standard basis is given by

$$T_{nst}(z) = -\beta(z)\partial\gamma(z) + \psi^\dagger(z)\partial\psi(z) + \bar{\psi}^\dagger(z)\partial\bar{\psi}(z) + \frac{1}{2}((i\partial\phi(z))^2 - (\partial\phi'(z))^2). \tag{3.16}$$

This energy–momentum tensor has no terms with background charges, and obeys the OPE

$$T_{nst}(z)T_{nst}(w) = \frac{2T_{nst}(w)}{(z-w)^2} + \frac{\partial T_{nst}(w)}{z-w}. \tag{3.17}$$

The screening currents in the non-standard basis are

$$\begin{aligned}
 s_{n,1}(z) &= \left(\psi^\dagger(z) + \frac{1}{2}\beta(z)\bar{\psi}(z) \right) \exp \left\{ -\frac{1}{\alpha_+}(i\phi(z) - \phi'(z)) \right\} \\
 s_{n,2}(z) &= \left(\bar{\psi}^\dagger(z) + \frac{1}{2}\beta(z)\psi(z) \right) \exp \left\{ -\frac{1}{\alpha_+}(i\phi(z) + \phi'(z)) \right\}.
 \end{aligned} \tag{3.18}$$

Similar results as (3.15), (3.18) have also been given in [16, 18], though based on different approaches and conventions.

3.3. $osp(2|2)_k^{(2)}$

Here $osp(2|2)$ is decomposed into $g_0 \oplus g_1$. The action of e^{A_t} with $A_t = xF + \theta f + \theta' f'$ on the highest weight state $|p, q', p\rangle$ generates a coherent state of $osp(2|2)$ in the basis given by (2.7), where x is bosonic coordinate, θ and θ' are fermionic coordinates. Again write

$$b_g e^{A_t} |p, q', p\rangle = D_g e^{A_t} |p, q', p\rangle \quad f_g e^{A_t} |p, q', p\rangle = d_g e^{A_t} |p, q', p\rangle \quad (3.19)$$

where b_g, f_g stand for the bosonic and fermionic generators of $osp(2|2)$ in the basis (2.7), and D_g, d_g are the corresponding differential operators. After some algebraic computations we get

$$\begin{aligned} D_F &= \partial_x \\ d_f &= \partial_\theta - \theta \partial_x \\ d_{f'} &= \partial_{\theta'} + \theta' \partial_x \\ D_H &= p + 2x \partial_x + \theta \partial_\theta + \theta' \partial_{\theta'} \\ D_{H'} &= q' - \theta \partial_{\theta'} - \theta' \partial_\theta \\ d_e &= p\theta + q'\theta' - x \partial_\theta + \theta x \partial_x + \theta \theta' \partial_{\theta'} \\ D_E &= -px + q'\theta \theta' - x^2 \partial_x - x\theta \partial_\theta - x\theta' \partial_{\theta'} \\ d_{e'} &= p\theta' + q'\theta + x \partial_{\theta'} + \theta' x \partial_x + \theta' \theta \partial_\theta. \end{aligned} \quad (3.20)$$

Indeed these differential operators satisfy the algebraic relations (2.7).

By means of the differential operator realization (3.20) and a substitution similar to (3.5) and an addition of suitable anomalous terms, we find the free field realization of $osp(2|2)_k^{(2)}$,

$$\begin{aligned} J_E(z) &= \beta(z) \\ j_e(z) &= \psi^\dagger(z) - \beta(z)\psi(z) \\ j_{e'}(z) &= \psi'^\dagger(z) + \beta(z)\psi'(z) \\ J_H(z) &= i\alpha_+ \partial\phi(z) + 2\beta(z)\gamma(z) + \psi(z)\psi^\dagger(z) + \psi'(z)\psi'^\dagger(z) \\ J_{H'}(z) &= \alpha_+ \partial\phi'(z) - \psi(z)\psi'^\dagger(z) - \psi'(z)\psi^\dagger(z) \\ j_f(z) &= -\alpha_+ (i\partial\phi(z)\psi(z) + \partial\phi'(z)\psi'(z)) + \gamma(z)\psi^\dagger(z) - \beta(z)\gamma(z)\psi(z) \\ &\quad - \psi(z)\psi'(z)\psi'^\dagger(z) - (2k+1)\partial\psi(z) \\ j_{f'}(z) &= -\alpha_+ (i\partial\phi(z)\psi'(z) + \partial\phi'(z)\psi(z)) - \gamma(z)\psi'^\dagger(z) - \beta(z)\gamma(z)\psi'(z) \\ &\quad - \psi'(z)\psi(z)\psi^\dagger(z) - (2k+1)\partial\psi'(z) \\ J_F(z) &= -\alpha_+ (i\partial\phi(z)\gamma(z) - \partial\phi'(z)\psi(z)\psi'(z)) - \beta(z)\gamma^2(z) \\ &\quad - \gamma(z)(\psi(z)\psi^\dagger(z) + \psi'(z)\psi'^\dagger(z)) - k\partial\gamma(z) \\ &\quad + (k+1)(\psi(z)\partial\psi(z) - \psi'(z)\partial\psi'(z)) \end{aligned} \quad (3.21)$$

where $\psi'(z)$ and $\psi'^\dagger(z)$ are free fermionic fields having the OPEs

$$\psi'(z)\psi'^\dagger(w) = \psi'^\dagger(z)\psi'(w) = -\frac{1}{z-w}. \quad (3.22)$$

It is straightforward to check that the above currents satisfy the OPEs of $osp(2|2)_k^{(2)}$ given in last section.

The energy–momentum tensor is

$$T_t(z) = -\beta(z)\partial\gamma(z) + \psi^\dagger(z)\partial\psi(z) + \psi'^\dagger(z)\partial\psi'(z) + \frac{1}{2}((i\partial\phi(z))^2 - (\partial\phi'(z))^2). \quad (3.23)$$

There are no background charges in the expression of the energy–momentum tensor, and its OPE reads

$$T_t(z)T_t(w) = \frac{2T_t(w)}{(z-w)^2} + \frac{\partial T_t(w)}{z-w}. \tag{3.24}$$

So we are dealing with a conformal field theory with zero Virasoro central charge.

It is first pointed out in [31] that in twisted case, the usual method to derive the screening currents is inappropriate. The screening currents should be twisted. The twisted screening currents for $osp(2|2)^{(2)}$ are found to be

$$\begin{aligned} s_{t,+}(z) &= (\psi^\dagger(z) + \psi'^\dagger(z) + \beta(z)\psi(z) - \beta(z)\psi'(z)) \exp\left\{-\frac{1}{\alpha_+}(i\phi(z) + \phi'(z))\right\} \\ &= (\psi^\dagger(z) + \psi'^\dagger(z) + \beta(z)\psi(z) - \beta(z)\psi'(z))\tilde{s}_{t,+}(z) \\ s_{t,-}(z) &= (\psi^\dagger(z) - \psi'^\dagger(z) + \beta(z)\psi(z) + \beta(z)\psi'(z)) \exp\left\{-\frac{1}{\alpha_+}(i\phi(z) - \phi'(z))\right\} \\ &= (\psi^\dagger(z) - \psi'^\dagger(z) + \beta(z)\psi(z) + \beta(z)\psi'(z))\tilde{s}_{t,-}(z). \end{aligned} \tag{3.25}$$

These screening currents satisfy the OPEs,

$$j_e(z)s_{t,\pm}(w) = J_E(z)s_{t,\pm}(w) = j_{\bar{e}}(z)s_{t,\pm}(w) = J_H(z)s_{t,\pm}(w) = J_{H'}(z)s_{t,\pm}(w) = \dots \tag{3.26}$$

and

$$\begin{aligned} j_f(z)s_{t,\pm}(w) &= -\partial_w \left(\frac{\alpha_+^2}{z-w} \tilde{s}_{t,\pm}(w) \right) \\ j_{f'}s_{t,\pm}(w) &= -\partial_w \left(\frac{\alpha_+^2}{z-w} \tilde{s}_{t,\pm}(w) \right) \\ J_F(z)s_{t,+}(w) &= \partial_w \left(\frac{\alpha_+^2}{z-w} (\psi(w) - \psi'(w))\tilde{s}_{t,+}(w) \right) \\ J_F(z)s_{t,-}(w) &= \partial_w \left(\frac{\alpha_+^2}{z-w} (\psi(w) + \psi'(w))\tilde{s}_{t,-}(w) \right). \end{aligned} \tag{3.27}$$

There does not seem to have screening current of the second kind for $osp(2|2)_k^{(2)}$.

4. Primary fields

Primary fields are fundamental objects in conformal field theories. A primary field Ψ has the following OPE with the energy–momentum tensor,

$$T(z)\Psi(w) = \frac{\Delta_\Psi}{(z-w)^2}\Psi(w) + \frac{\partial_w \Psi(w)}{z-w} + \dots \tag{4.1}$$

where the Δ_Ψ is the conformal dimension of Ψ . Moreover, the OPEs of Ψ with the affine currents do not contain poles higher than first order. A special kind of the primary fields is highest weight state.

Let us remark that certain representations were investigated for $osp(2|2)$ in [26, 27] and for $osp(2|2)^{(1)}$ in [28, 29]. Here we are concerned with primary fields, which requires the construction of all representations.

4.1. $osp(2|2)^{(1)}$ primary fields in the standard basis

It is easy to see that the highest weight state of the algebra is

$$V_{p,q}(z) = \exp \left\{ \frac{2}{\alpha_+} (p i\phi(z) - q\phi'(z)) \right\} \tag{4.2}$$

where p, q are given complex numbers labelling the representation. The conformal dimension of the field is

$$\Delta_{p,q} = \frac{p(p+1) - q(q+1)}{k+1}. \tag{4.3}$$

If $q \neq p, -p-1$, then $\Delta_{p,q} \neq 0$ and the corresponding representations are typical. When $q = p, -p-1$, we have $\Delta_{p,q} = 0$ and atypical representations arise. In order for the representation to be finite dimensional, we find that p must be an integer or half-integer. The full bases of the representation labelled by p, q are

$$\begin{aligned} S_{p,q}^m(z) &= (-\gamma(z))^{p-m} V_{p,q}(z) & m = p, p-1, \dots, -(p-1), -p \\ s_{p,q}^n(z) &= (-\gamma(z))^{(p-3/2)-n} (\bar{\psi}(z) + \frac{1}{2}\gamma(z)\psi(z)) V_{p,q}(z) \\ & n = (p-3/2), \dots, -(p+1/2) & p \geq 1/2 \\ \tilde{s}_{p,q}^l(z) &= (p-q)(-\gamma(z))^{(p-3/2)-l} (\bar{\psi}(z) - \frac{1}{2}\gamma(z)\psi(z)) V_{p,q}(z) \\ & l = (p-3/2), \dots, -(p+3/2) & p \geq 3/2 \\ \phi_{p,q}(z) &= (p-q)\psi(z) V_{p,q}(z) \\ S_{p,q}^s(z) &= (p-q)(-\gamma(z))^{(p-2)-s} \psi(z)\bar{\psi}(z) V_{p,q}(z) \\ & s = (p-2), \dots, -(p+2) & p \geq 2. \end{aligned} \tag{4.4}$$

The dimensions of $S_{p,q}^m(z)$ and $s_{p,q}^n(z)$ are $2p+1$ and $2p$, respectively. On the other hand both $\tilde{s}_{p,q}^l(z)$ and $S_{p,q}^s(z)$ have $(2p+1)$ independent components. Note that $\phi_{p,q}(z)$ is one dimensional. So the dimension of a typical representation (where $q \neq p, -p-1$) is $8p+4$. For an atypical representation corresponding to $q = p, -p-1$, $S_{p,q}^m(z)$ and $s_{p,q}^n(z)$ are the only non-vanishing fields and so the dimension of the atypical representation is $4p+1$.

By means of the free field representations given in section 3, we compute the OPEs of affine currents with the primary fields. The OPEs of the $osp(2|2)$ currents with $S_{p,q}^m(z)$ are

$$\begin{aligned} J_E(z)S_{p,q}^m(w) &= \frac{p-m}{z-w} S_{p,q}^{m+1}(w) \\ J_F(z)S_{p,q}^m(w) &= \frac{p+m}{z-w} S_{p,q}^{m-1}(w) \\ J_H(z)S_{p,q}^m(w) &= \frac{2m}{z-w} S_{p,q}^m(w) \\ J_{H'}(z)S_{p,q}^m(w) &= \frac{2q}{z-w} S_{p,q}^m(w) \\ j_e(z)S_{p,q}^m(w) &= 0 \\ j_{\bar{e}}(z)S_{p,q}^m(w) &= 0 \\ j_f(z)S_{p,q}^m(w) &= \frac{1}{z-w} ((m-q)s_{p,q}^{m-1/2}(w) - \tilde{s}_{p,q}^{m-1/2}(w)) & m \leq (p-1) \\ j_{\bar{f}}(z)S_{p,q}^p(w) &= \frac{-1}{z-w} \phi_{p,q}(w) \\ j_{\bar{f}}(z)S_{p,q}^m(w) &= \frac{1}{z-w} ((p+m)s_{p,q}^{m-3/2}(w) - \tilde{s}_{p,q}^{m-3/2}(w)). \end{aligned} \tag{4.5}$$

When $q = p$, terms involving $\phi_{p,q}(z)$ and $\tilde{s}_{p,q}^l(z)$ disappear. The OPEs with $s_{p,q}^n(z)$ are

$$\begin{aligned}
 J_E(z)s_{p,q}^n(w) &= \frac{(p-3/2)-n}{z-w}s_{p,q}^{n+1}(w) \\
 J_F(z)s_{p,q}^n(w) &= \frac{(p+1/2)+n}{z-w}s_{p,q}^{n-1}(w) \\
 J_H(z)s_{p,q}^n(w) &= \frac{2n+2}{z-w}s_{p,q}^n(w) \\
 J_{H'}(z)s_{p,q}^n(w) &= \frac{1+2q}{z-w}s_{p,q}^n(w) \\
 j_e(z)s_{p,q}^n(w) &= -\frac{1}{z-w}S_{p,q}^{n+1/2}(w) \\
 j_{\bar{e}}(z)s_{p,q}^n(w) &= -\frac{1}{z-w}S_{p,q}^{n+3/2}(w) \\
 j_f(z)s_{p,q}^n(w) &= -\frac{1}{z-w}S_{p,q}^{n-1/2}(w) \\
 j_{\bar{f}}(z)s_{p,q}^n(w) &= -\frac{1}{z-w}S_{p,q}^{n-3/2}(w).
 \end{aligned} \tag{4.6}$$

When $q = p$, terms containing $S_{p,q}^s(w)$ disappear. The relations of the currents with $\tilde{s}_{p,q}^l(z)$ and $S_{p,q}^s(w)$ are

$$\begin{aligned}
 J_E(z)\tilde{s}_{p,q}^l(w) &= \frac{1}{z-w}(((p-1/2)-l)\tilde{s}_{p,q}^{l+1}(w) - (p-q)s_{p,q}^{l+1}(w)) \quad l \leq (p-5/2) \\
 J_E(z)\tilde{s}_{p,q}^{p-3/2}(w) &= \frac{1}{z-w}\phi_{p,q}(w) \\
 J_F(z)\tilde{s}_{p,q}^l(w) &= \frac{(p+3/2)+l}{z-w}\tilde{s}_{p,q}^{l-1}(w) \\
 J_H(z)\tilde{s}_{p,q}^l(w) &= \frac{2l+2}{z-w}\tilde{s}_{p,q}^l(w) \\
 J_{H'}(z)\tilde{s}_{p,q}^l(w) &= \frac{1+2q}{z-w}\tilde{s}_{p,q}^l(w) \\
 j_e(z)\tilde{s}_{p,q}^l(w) &= 0 \\
 j_{\bar{e}}(z)\tilde{s}_{p,q}^l(w) &= -\frac{p-q}{z-w}S_{p,q}^{l+3/2}(w) \\
 j_f(z)\tilde{s}_{p,q}^l(w) &= \frac{(q-1/2)-l}{z-w}S_{p,q}^{l-1/2}(w) \\
 j_{\bar{f}}(z)\tilde{s}_{p,q}^l(w) &= -\frac{(p+3/2)+l}{z-w}S_{p,q}^{l-3/2}(w)
 \end{aligned} \tag{4.7}$$

and

$$\begin{aligned}
 J_E(z)S_{p,q}^s(w) &= \frac{(p-2)-s}{z-w}S_{p,q}^{s+1}(w) \\
 J_F(z)S_{p,q}^s(w) &= \frac{(p+2)+s}{z-w}S_{p,q}^{s-1}(w) \\
 J_H(z)S_{p,q}^s(w) &= \frac{2s+4}{z-w}S_{p,q}^s(w) \\
 J_{H'}(z)S_{p,q}^s(w) &= \frac{2+2q}{z-w}S_{p,q}^s(w)
 \end{aligned}$$

$$\begin{aligned}
j_e(z)S_{p,q}^s(w) &= \frac{1}{z-w} \bar{s}_{p,q}^{s+1/2}(w) \\
j_{\bar{e}}(z)S_{p,q}^s(w) &= \frac{1}{z-w} (\bar{s}_{p,q}^{s+3/2}(w) - (p-q)s_{p,q}^{s+3/2}(w)) \quad s \leq (p-3) \\
j_{\bar{e}}(z)S_{p,q}^{p-2}(w) &= \frac{1}{z-w} \phi_{p,q}(w) \\
j_f(z)S_{p,q}^s(w) &= 0 \\
j_{\bar{f}}(z)S_{p,q}^s(w) &= 0.
\end{aligned} \tag{4.8}$$

Finally OPEs involving $\phi_{p,q}(w)$ read

$$\begin{aligned}
J_E(z)\phi_{p,q}(w) &= \dots \\
J_F(z)\phi_{p,q}(w) &= \frac{1}{z-w} ((2p+1)\bar{s}_{p,q}^{p-3/2}(w) - 2p(p-q)s_{p,q}^{p-3/2}(w)) \\
J_H(z)\phi_{p,q}(w) &= \frac{2p+1}{z-w} \phi_{p,q}(w) \\
J_{H'}(z)\phi_{p,q}(w) &= \frac{1+2q}{z-w} \phi_{p,q}(w) \\
j_e(z)\phi_{p,q}(w) &= \frac{p-q}{z-w} S_{p,q}^p(w) \\
j_{\bar{e}}(z)\phi_{p,q}(w) &= 0 \\
j_f(z)\phi_{p,q}(w) &= 0 \\
j_{\bar{f}}(z)\phi_{p,q}(w) &= -\frac{(p+q)+1}{z-w} S_{p,q}^{p-2}(w).
\end{aligned} \tag{4.9}$$

4.2. $osp(2|2)^{(1)}$ primary fields in the non-standard basis

The highest weight state of the algebra is

$$V_{J,q}(z) = \exp \left\{ \frac{2}{\alpha_+} (Ji\phi(z) - q\phi'(z)) \right\} \tag{4.10}$$

where J, q are given complex numbers specifying the representation. The conformal dimension of the field is

$$\Delta_{J,q} = \frac{J^2 - q^2}{k+1}. \tag{4.11}$$

If $q \neq \pm J$, then $\Delta_{J,q} \neq 0$ and the corresponding representations are typical. When $q = \pm J$, atypical representations arise. For the representation to be finite dimensional, it turns out that J must be an integer or half-integer and moreover if $J = 0$ then q must also be zero. For $J = 0 = q$, the atypical representation is obviously one dimensional. For $J \neq 0$, a representation labelled by J, q has the following bases:

$$\begin{aligned}
N_{J,q}^m(z) &= [2J(-\gamma(z))^{J-m} - q(J-m)(-\gamma(z))^{J-m-1} \psi(z) \bar{\psi}(z)] V_{J,q}(z) \\
&\quad m = J, J-1, \dots, -(J-1), -J, J \geq 1/2 \\
n_{J,q}^l(z) &= (J-q)(-\gamma(z))^{(J-1/2)-l} \psi(z) V_{J,q}(z) \\
&\quad l = (J-1/2), \dots, -(J-1/2) \quad J \geq 1/2
\end{aligned}$$

$$\begin{aligned}
 \bar{n}_{J,q}^l(z) &= (J+q)(-\gamma(z))^{(J-1/2)-l} \bar{\psi}(z) V_{J,q}(z) \\
 &\quad l = (J-1/2), \dots, -(J-1/2) \quad J \geq 1/2 \\
 \mathcal{N}_{J,q}^n(z) &= (J^2 - q^2)(-\gamma(z))^{(J-1)-l} \psi(z) \bar{\psi}(z) V_{J,q}(z) \\
 &\quad n = (J-1), \dots, -(J-1) \quad J \geq 1.
 \end{aligned}
 \tag{4.12}$$

It is easy to see that $N_{J,q}^m(z)$ and $\mathcal{N}_{J,q}^n(z)$ have $(2J+1)$ and $(2J-1)$ independent components, respectively, and the dimensions of $n_{J,q}^l(z)$ and $\bar{n}_{J,q}^l(z)$ are both $2J$. So the dimension of a typical representation (where $q \neq \pm J$) is $8J$. For an atypical representation, either only $N_{J,q}^m(z)$ and $n_{J,q}^n(z)$ survive (when $q = -J$) or only $N_{J,q}^m(z)$ and $\bar{n}_{J,q}^n(z)$ remain (when $q = J$). So the dimension of the atypical representation is $4J+1$.

The OPEs of the $osp(2|2)$ currents with $N_{J,q}^m(z)$ are

$$\begin{aligned}
 J_E(z)N_{J,q}^m(w) &= \frac{J-m}{z-w} N_{J,q}^{m+1}(w) \\
 J_F(z)N_{J,q}^m(w) &= \frac{J+m}{z-w} N_{J,q}^{m-1}(w) \\
 J_H(z)N_{J,q}^m(w) &= \frac{2m}{z-w} N_{J,q}^m(w) \\
 J_{H'}(z)N_{J,q}^m(w) &= \frac{2q}{z-w} N_{J,q}^m(w) \\
 j_e(z)N_{J,q}^m(w) &= -\frac{J-m}{z-w} \bar{n}_{J,q}^{m+1/2}(w) \\
 j_{\bar{e}}(z)N_{J,q}^m(w) &= -\frac{J-m}{z-w} n_{J,q}^{m+1/2}(w) \\
 j_f(z)N_{J,q}^m(w) &= \frac{J+m}{z-w} n_{J,q}^{m-1/2}(w) \\
 j_{\bar{f}}(z)N_{J,q}^m(w) &= \frac{J+m}{z-w} \bar{n}_{J,q}^{m-1/2}(w).
 \end{aligned}
 \tag{4.13}$$

We see that $n_{J,q}^l(z)$ and $\bar{n}_{J,q}^l(z)$ are generated from $N_{J,q}^m(z)$ by the action of the fermionic currents. The OPEs involving $n_{J,q}^l(z)$ are

$$\begin{aligned}
 J_E(z)n_{J,q}^l(w) &= \frac{(J-1/2)-l}{z-w} n_{J,q}^{l+1}(w) \\
 J_F(z)n_{J,q}^l(w) &= \frac{(J-1/2)+l}{z-w} n_{J,q}^{l-1}(w) \\
 J_H(z)n_{J,q}^l(w) &= \frac{2l}{z-w} n_{J,q}^l(w) \\
 J_{H'}(z)n_{J,q}^l(w) &= -\frac{1+2q}{z-w} n_{J,q}^l(w) \\
 j_e(z)n_{J,q}^l(w) &= \frac{-1}{z-w} \left(\frac{J-q}{2J} N_{J,q}^{l+1/2}(w) - \frac{(J-1/2)-l}{2J} \mathcal{N}_{J,q}^{l+1/2}(w) \right) \\
 j_{\bar{e}}(z)n_{J,q}^l(w) &= 0 \\
 j_f(z)n_{J,q}^l(w) &= 0 \\
 j_{\bar{f}}(z)n_{J,q}^l(w) &= \frac{-1}{z-w} \left(\frac{J-q}{2J} N_{J,q}^{l-1/2}(w) + \frac{(J-1/2)+l}{2J} \mathcal{N}_{J,q}^{l-1/2}(w) \right)
 \end{aligned}
 \tag{4.14}$$

and

$$\begin{aligned}
J_E(z)\bar{n}_{J,q}^l(w) &= \frac{(J-1/2)-l}{z-w}\bar{n}_{J,q}^{l+1}(w) \\
J_F(z)\bar{n}_{J,q}^l(w) &= \frac{(J-1/2)+l}{z-w}\bar{n}_{J,q}^{l-1}(w) \\
J_H(z)\bar{n}_{J,q}^l(w) &= \frac{2l}{z-w}\bar{n}_{J,q}^l(w) \\
J_{H'}(z)\bar{n}_{J,q}^l(w) &= \frac{1+2q}{z-w}\bar{n}_{J,q}^l(w) \\
j_e(z)\bar{n}_{J,q}^l(w) &= 0 \\
j_{\bar{e}}(z)\bar{n}_{J,q}^l(w) &= \frac{-1}{z-w}\left(\frac{J+q}{2J}N_{J,q}^{l+1/2}(w) + \frac{(J-1/2)-l}{2J}N_{J,q}^{l+1/2}(w)\right) \\
j_f(z)\bar{n}_{J,q}^l(w) &= \frac{-1}{z-w}\left(\frac{J+q}{2J}N_{J,q}^{l-1/2}(w) - \frac{(J-1/2)+l}{2J}N_{J,q}^{l-1/2}(w)\right) \\
j_{\bar{f}}(z)\bar{n}_{J,q}^l(w) &= 0.
\end{aligned} \tag{4.15}$$

Finally, the OPEs of the currents with $\mathcal{N}_{J,q}^n(z)$ are

$$\begin{aligned}
J_E(z)\mathcal{N}_{J,q}^n(w) &= \frac{(J-1)-n}{z-w}\mathcal{N}_{J,q}^{n+1}(w) \\
J_F(z)\mathcal{N}_{J,q}^n(w) &= \frac{(p-1)+n}{z-w}\mathcal{N}_{J,q}^{n-1}(w) \\
J_H(z)\mathcal{N}_{J,q}^n(w) &= \frac{2n}{z-w}\mathcal{N}_{J,q}^n(w) \\
J_{H'}(z)\mathcal{N}_{p,q}^n(w) &= \frac{2q}{z-w}\mathcal{N}_{p,q}^n(w) \\
j_e(z)\mathcal{N}_{J,q}^n(w) &= -\frac{J-q}{z-w}\bar{n}_{J,q}^{n+1/2}(w) \\
j_{\bar{e}}(z)\mathcal{N}_{J,q}^n(w) &= \frac{J+q}{z-w}n_{J,q}^{n+1/2}(w) \\
j_f(z)\mathcal{N}_{p,q}^n(w) &= \frac{J+q}{z-w}n_{p,q}^{n-1/2}(w) \\
j_{\bar{f}}(z)\mathcal{N}_{p,q}^n(w) &= -\frac{J-q}{z-w}\bar{n}_{p,q}^{n-1/2}(w).
\end{aligned} \tag{4.16}$$

We would like to make a remark on the special case when $J = 0, q \neq 0$. In this case, the representation with the highest weight state (4.10) is typical. However, this representation is infinite dimensional, as is seen from the following bases of the representation:

$$\begin{aligned}
N_{q,\pm}^m(z) &= \left(\gamma^{-m}(z) \mp \frac{m}{2}\gamma^{-m-1}(z)\psi(z)\bar{\psi}(z)\right)V_{0,q}(z) \quad m = 0, -1, -2, \dots \\
n_q^l(z) &= \gamma^{(-l-1/2)}(z)\psi(z)V_{0,q}(z) \quad l = -1/2, -3/2, \dots \\
\bar{n}_q^l(z) &= \gamma^{(-l-1/2)}(z)\bar{\psi}(z)V_{0,q}(z) \quad l = -1/2, -3/2, \dots \\
\mathcal{N}_q^n(z) &= \gamma^{(-nJ-1)}\psi(z)\bar{\psi}(z)V_{0,q}(z) \quad n = -1, -2, \dots
\end{aligned} \tag{4.17}$$

4.3. $osp(2|2)^{(2)}$ primary fields

The highest weight state of the algebra is

$$V_{J,\pm}(z) = \exp \left\{ \frac{2}{\alpha_+} J(i\phi(z) \pm \phi'(z)) \right\} \tag{4.18}$$

where J is any given complex number characterizing the representation. The conformal dimension of the field is

$$\Delta_{J,\pm} = 0. \tag{4.19}$$

So there are no typical representations and all representations are atypical. It turns out that for the representation to be finite dimensional, J has to be an integer or half-integer. The full bases of the representation are

$$\begin{aligned} \mathcal{T}_{J,\pm}^m(z) &= [(-\gamma(z))^{J-m} \mp (J-m)(-\gamma(z))^{J-m-1}\psi(z)\psi'(z)]V_{J,\pm}(z) \\ m &= J, J-1, \dots, -(J-1), -J \end{aligned} \tag{4.20}$$

$$t_{J,\pm}^l(z) = (-\gamma(z))^{(J-1/2)-l}(\psi(z) \mp \psi'(z))V_{J,q}(z) \quad l = (J-1/2), \dots, -(J-1/2).$$

$\mathcal{T}_{J,\pm}^m(z)$ and $t_{J,\pm}^l(z)$ have $(2J+1)$ and $2J$ independent components, respectively. So the dimension of the representation is $4J+1$.

The OPEs of the currents with $\mathcal{T}_{J,\pm}^m(z)$ are

$$\begin{aligned} J_E(z)\mathcal{T}_{J,\pm}^m(w) &= \frac{J-m}{z-w}\mathcal{T}_{J,\pm}^{m+1}(w) \\ J_F(z)\mathcal{T}_{J,\pm}^m(w) &= \frac{J+m}{z-w}\mathcal{T}_{J,\pm}^{m-1}(w) \\ J_H(z)\mathcal{T}_{J,\pm}^m(w) &= \frac{2m}{z-w}\mathcal{T}_{J,\pm}^m(w) \\ J_{H'}(z)\mathcal{T}_{J,\pm}^m(w) &= \frac{\mp 2J}{z-w}\mathcal{T}_{J,\pm}^m(w) \\ j_e(z)\mathcal{T}_{J,\pm}^m(w) &= -\frac{J-m}{z-w}t_{J,q\pm}^{m+1/2}(w) \\ j_{e'}(z)\mathcal{T}_{J,\pm}^m(w) &= \pm\frac{J-m}{z-w}t_{J,\pm}^{m+1/2}(w) \\ j_f(z)\mathcal{T}_{J,\pm}^m(w) &= -\frac{J+m}{z-w}t_{J,\pm}^{m-1/2}(w) \\ j_{f'}(z)\mathcal{T}_{J,\pm}^m(w) &= \pm\frac{J+m}{z-w}t_{J,\pm}^{m-1/2}(w). \end{aligned} \tag{4.21}$$

We see that $t_{J,\pm}^l(z)$ are generated from $\mathcal{T}_{J,\pm}^m(z)$ by the action of the fermionic currents. The OPEs involving $t_{J,\pm}^l(z)$ are

$$\begin{aligned} J_E(z)t_{J,\pm}^l(w) &= \frac{(J-1/2)-l}{z-w}t_{J,\pm}^{l+1}(w) \\ J_F(z)t_{J,\pm}^l(w) &= \frac{(J-1/2)+l}{z-w}t_{J,\pm}^{l-1}(w) \\ J_H(z)t_{J,\pm}^l(w) &= \frac{2l}{z-w}t_{J,\pm}^l(w) \\ J_{H'}(z)t_{J,\pm}^l(w) &= \frac{1+2J}{z-w}t_{J,\pm}^l(w) \end{aligned}$$

$$\begin{aligned}
j_e(z)t_{J,\pm}^l(w) &= \frac{-1}{z-w} \mathcal{T}_{J,\pm}^{l+1/2}(w) \\
j_{e'}(z)t_{J,\pm}^l(w) &= \frac{\pm 1}{z-w} \mathcal{T}_{J,\pm}^{l+1/2}(w) \\
j_f(z)t_{J,\pm}^l(w) &= \frac{1}{z-w} \mathcal{T}_{J,\pm}^{l-1/2}(w) \\
j_{f'}(z)t_{J,q}^l(w) &= \frac{\pm 1}{z-w} \mathcal{T}_{J,\pm}^{l-1/2}(w).
\end{aligned}
\tag{4.22}$$

5. Conclusions

We have studied the conformal field theories associated with the current superalgebras $osp(2|2)^{(1)}$ and $osp(2|2)^{(2)}$. We construct the free field representations and screen currents of these two superalgebras at general level k . We also construct the primary fields corresponding to both typical and atypical representations. Both conformal field theories have vanishing central charges. In the case of $osp(2|2)^{(1)}$, there exists an infinite family of negative dimensional primary operators so that the corresponding conformal field theory is non-unitary. For the case of $osp(2|2)^{(2)}$, the dimension of all primary fields vanishes and so they all correspond to atypical representations of the current superalgebra. Our results provide a useful algebraic background in the study of disordered systems using the supersymmetric method, which will be investigated elsewhere.

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